from the assumption of constant density and homentropy. So the solution, which satisfies Eq. (5) without changing the assumption, has physically no sense. On the other hand, it is not unnatural to think that the flowfield in the shock layer has the same pattern as the flow with the boundary condition that $v \to 0$ at infinity, since in two-dimensional flow the semicircular piston expands in a fluid at rest.

With regard to the last statement of Roe's Comment, we believe that even when the analysis is improved by taking the density and entropy changes into consideration, if the density and entropy do not change very much (as would be true in practice), the flow in the self-consistent region will also not be changed very much.

We would like to express our appreciation to Roe for his Comment. We wish to conclude by saying that although our analysis obviously has some defects, it is not a complete one, but it is the first for higher approximations.

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Re-Examination of the Role of Radiation in Hybrid Regression Rate Theory

P. J. Paul*
Indian Institute of Science, Bangalore, India

SOME studies on hybrid combustion theory have been reported recently by Rastogi et al.[1,2] The principal conclusion of these two papers is that radiant flux is very important in the operating regime of hybrid rockets. In these studies the lower limit of the oxidizer fluxes at which experiments have been conducted is about 1.5 to 1.8 gm/cm^2-sec. Wooldrige and Muzzy[3] show that for most cases of interest, the radiant flux is important only at oxidizer fluxes of the order 0.2 to 0.4 gm/cm^2-sec. Hence one should not expect radiant flux to be important in the experimental regime of the authors.[1,2] After a careful examination of the experimental results reported by Rastogi et al.,[1,2] it appears that their conclusions are incorrect. This Comment is intended to point out the sources of the errors and possible corrective procedures.

The authors[1] consider the regression of a tubular grain through the port of which oxidizer flows. The balance of the heat flux at the surface of the fuel is written as (notations same as in Ref. 1)

$$\rho_f \dot{r} \Delta H = \dot{Q} - \dot{Q}_{re} \qquad \text{[Eqs. (14) of Ref. 1]}$$

where

$$\dot{Q}_{re} = \sigma\, \epsilon_w \epsilon_r T_r^4 \qquad \text{[Eq. (16) of Ref. 1]}$$

and

$$\dot{Q}_c = \text{convective flux}$$

It is stated that $\dot{Q}_r$, the radiant flux to the wall (it is not clear if the wall refers to fuel surface, but is presumed so), is neglected. The expression of $\dot{Q}_{re}$ is obtained from the work of Wooldrige and Muzzy[3] in which it is clearly and correctly identified as the radiant flux emitted by the flame and absorbed by the surface. It has to be added to the convective flux and *not* subtracted as is done in Eq. (14) of Ref. 1; however it has to be suitably reduced to account for the blocking effect. In their analysis they assume that radiant flux is zero at time $t=0$ and convective flux is present. It is not clear why one of these fluxes is zero at $t=0$ and as to why time should enter into the picture in this way in a steady-state problem. The heat balance at the wall should be considered under steady conditions, and when so done, both the terms will be present all the time.

Consequent upon their formulation the authors[1] obtain the relation

$$\log[\rho_f \Delta H/\sigma\epsilon_w p^\beta T_r^4(\dot{r}_t - \dot{r}_c) + 1] = \alpha\, pz \text{[Eq. (23) of Ref. 1]}$$

where z is identified as the diameter of the duct at any time t. It can be observed that the left-hand side (LHS) of this equation is zero at $t=0$ (since $\dot{r}_t = \dot{r}_0$) whereas the right hand side (RHS) has a nonzero value. The equation is therefore inconsistent and hence incorrect. The fact that the authors obtained a nonzero value of $(\dot{r}_t - \dot{r}_c)$ at $t=0$ (Fig. 12 of Ref. 1) raises doubts about their calculation procedure.

The experimental data on regression rate are used to obtain the LHS of Eq. (23) of Ref. 1 for various values of z. The linearity of the plot of LHS vs z of Eq. (23) is taken as a support for the theory. Unfortunately Eq. (23) demands that the plot, LHS vs z, must go through (0,0). As seen from Fig. 11 of Ref. 1 this does not happen.

Finally the term $\dot{Q}_{re}$ with the interpretation that it represents the heat transfer from the flame front to the exhaust gases is irrelevant to the heat balance at the wall. In fact correct heat balance at the wall is[3]

$$\dot{Q}_w = \dot{Q}_c f + \dot{Q}_r \tag{1}$$

where

$$\dot{Q}_r = \sigma\epsilon_w p^\beta \epsilon_r T_r^4 (1 - e^{-\alpha p z})$$

and f is the factor to reduce the convective flux to account for blowing.

The next point is concerned with the representation of the variation of radius with time as[1]

$$r = A + kt^x \qquad \text{[Eq. (25) of Ref. 1]}$$

which gives

$$\dot{r} = kxt^{x-1} \qquad \text{[Eq. (26) of Ref. 1]}$$

It must be noted that both A and k are positive. The range of values of x lies between 0 and 1. x cannot be greater than 1 since this implies that regression rate increases with time, a fact which is contrary to observations (Fig. 2 of Ref. 1). If $x<1$, the regression rate is infinity at $t=0$. However, the plot in Fig. 2 of Ref. 1 shows a finite value for the regression rate at $t=0$. This implies that the curve fit is not appropriate, at least around $t=0$.

One of the aspects of the analysis of regression data is that the relation of curve fit should be given careful attention. The best technique is to presume a relation which has a theoretical basis. For instance in the case studied by Rastogi et al.[1] one can obtain from $\dot{r} = aG^n$ the following approximate relation for r vs t as

$$r^{1+2n} - r_0^{1+2n} = bt \tag{2}$$

Now a suitable curve fit treating n and b as unknowns will lead to values which will not have any singularities. Using the data of Ref. 1 given in Figs. 2 and 12 a replot of r^2 vs t is made and is shown in Fig. 1. It is very nearly a straight line which implies that n is about 0.5, which is true for laminar boundary layers. Equation (2) can be further refined to take

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*Research Fellow, Dept. of Aeronautical Engineering.

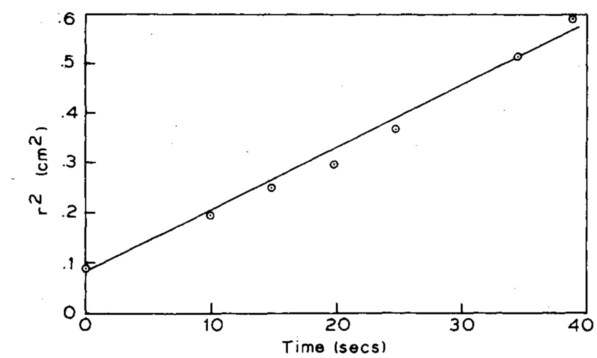

Fig. 1 Plot of the (radius)^2 vs time from the experiment of Ref. 1.

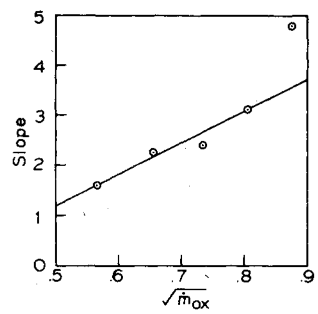

Fig. 2 Plot of the slope with $\sqrt{\dot m_{ox}}$ from data of Ref. 2.

account of 0/F variation. These methods of refining the curve fit are not discussed here.

Figure 9 (Ref. 1), which shows the plot of log a vs log n with time, indicates that a and n vary widely with time. This cannot be correct if the convective relation is meaningful. It is very likely that the data processing technique leading to a and n is incorrect.

Reference 2 treats the same problem in the case of aluminized propellants. This reference seems to use fundamental relations which are different from those in Ref. 1 and which are again incorrect. The heat balance relation is written as

$$\dot Q_w = \dot Q_T - (\dot Q_{re} + \dot Q_p)$$

where $\dot Q_T$ is taken as total heat produced at the flame zone and $\dot Q_p$ is the heat taken away by the particulate combustion products. In the event $\dot Q_{re} + \dot Q_p = 0$, which happens to occur at $t=0$ in the formulation[2] (a fact which should again be pointed out as being unclear), one obtains $\dot Q_w = \dot Q_T$.

The total heat produced at the flame zone written in terms of fluxes is $\rho_f \dot r H$ where H is the heat of combustion. Thus we obtain for the case of zero particle content

$$\dot Q_w = \rho_f \dot r h_{\text{eff}} = \rho_f \dot r H$$

a relation which leads to the absurdity that $h_{\text{eff}} = H$.

The implication is that term $\dot Q_T$ should not be the total heat released at the flame zone, but the heat transferred to the surface by convection and radiation itself. Also, as pointed out earlier, $\dot Q_{re}$ is irrelevant to the heat balance relation. Again the correct heat balance relation to be used is the one given by Eq. (1).

A few other features of this paper which are different from Ref. 1 though equally incorrect are as follows. The linearity of $(\dot r_0 - \dot r_t)$ vs $1/r$ plot and increased slope with oxidizer flow rate are taken as support for the predominance of radiation. In fact the authors conclude that with increased mass flow rate contribution to radiant heat flux increases. The reason given is that the flame temperature increases with mass flow rate. First, the flame temperature has no relation to the mass flow rate. In laminar boundary layer (which appears to be more relevant in the present case) the flame gets located at the position where stoichiometry of fuel and oxidizer is established and the flame temperature is adiabatic flame temperature at all flow rates.[4] The increase in oxidizer flow rate only affects the flame position and not the flame temperature. Second, the statement that radiant flux increases with mass flow rate is unacceptable in the view of evidence available in other studies.[3]

Third, the linearity of the plot of $(\dot r_0 - \dot r_t)$ vs $1/r$ and the increased slope with mass flow rate is simply explained using the convective theory as will be demonstrated.

We have $\dot r_t = aG^n$, from which we obtain $\dot r_0 - \dot r_t = a\,G^n - a\,G^{0n}$ where G^0 is the mass flux at $t=0$. Further if we take $G^0 \simeq 4\dot m_{ox}/\pi d^2$, a fact which is valid for large $0/F$, we obtain

$$\dot r_0 - \dot r_t = a\left[\frac{4\dot m_{ox}}{\pi d_0^2}\right]^n - a\left[\frac{4\dot m_{ox}}{\pi d^2}\right]^n = a\left[\frac{4\dot m_{ox}}{\pi}\right]^n\left[\frac{1}{d_0^{2n}} - \frac{1}{d^{2n}}\right]$$

If $n=0.5$ for the laminar flow conditions one obtains a linear fit for $(\dot r_0 - \dot r_t)$ vs $1/r$ data. The slope of the curve fit is proportional to $\dot m_{ox}$. Thus, the increase in slope can be explained as due to the increase in $\dot m_{ox}$. To further substantiate this, the slopes of the curves in Fig. 8 of Ref. 2 were plotted against $\dot m_{ox}$. This plot appears in Fig. 2. Though the linearity of four of the five data points may be taken as support for the convective theory the present author hesitates to draw this conclusion in view of the possible incorrectness in the analysis leading to regression rates.

Summarizing, the essential conclusions drawn by the authors,[1,2] that radiant flux is a very important phenomenon in hybrid combustion, is a consequence of a combination of incorrect mathematics and implausible physics. The role of radiant flux in hybrid combustion has already been well elucidated by Wooldridge and Muzzy.[3]

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Reply by Authors to P. J. Paul

R. P. Rastogi* and Desh Deepak†
University of Gorakhpur, Gorakhpur, India

THE comments of Paul on our recent papers on hybrid combustion[1,2] seem to originate from hasty conclusions due to lack of understanding of the basic theme of these contributions which has been emphasized too often in the relevant papers. These papers[1,2] stress the importance of radiative heat transfer to the combustion products and

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*Senior Professor of Chemistry, Department of Chemistry. Associate Fellow AIAA.

‡Senior Research Fellow, Department of Chemistry.